Reply by Authors to W. C. Robison and J. R. Nelson

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The data presented by Robison and Nelson in the preceding comment are one of six sets given by Nelson in Ref. 2 of the comment. We have examined all six sets hoping to apply the concept of generalized choking to them. However, the outlet pressure into which the mixing-tube flow discharged was, in all cases, atmospheric pressure. The initial stagnation pressure of the driven stream was also, in all cases, atmospheric pressure. The initial stagnation pressure of the driving stream did not exceed 5.5 atm. Under these conditions we cannot be sure that the outlet pressure was low enough to permit generalized choking to occur.

We have a few small indications that the methods of calculation derived for generalized choking may apply in some cases even when the back pressure is too high for the choking actually to occur; however, these indications are not well enough established for us to be ready to apply the methods of generalized choking to the data of Nelson. All of the six sets of data show a pressure rise in the downstream portion of the mixing tube. This pressure rise probably indicates that an extended shock (pseudo shock) is being formed in the driving stream. Because the mixing tube is short, the extended shock is not completed before the flow reaches the end of the tube. If the downstream pressure were lowered, this shock probably would be pulled out of the mixing tube; and soon after that we would expect generalized choking to occur. The evidence of choking is, of course, that further lowering of the downstream pressure produces no change upstream from the plane of choking.

Robison and Nelson, in their Fig. 2, show an interesting situation in which the driving stream, after entering the mixing tube, first expands and then contracts. The space remaining for the driven gas has the form of a converging-diverging nozzle, and the wall pressures in Fig. 1 of the comment indicate that the driven stream very well may become sonic at its narrowest cross section and supersonic as it flows downstream. We are not prepared to say whether the total flow in Fig. 2 of Robison and Nelson is choked or not. Even if we accept the flow of the driven stream as sonic at its narrowest cross section, it is still possible that a change in outlet pressure would cause a readjustment of the driving and driven streams and that this readjustment would change the area of the narrowest cross section of the driven stream. In such a case, downstream conditions and upstream conditions would not be decoupled, and the flow would not be truly choked.

The situation shown in Fig. 2 of the comment shows a substantial departure from one dimensionality; the driving gas has a strong divergence as it enters the mixing tube. Also, if we traverse the driving stream at its first cross-section maximum, we cross several interference fringes. Presumably, these fringes indicate that the density and the pressure at the axis of the driving stream are lower than at the edges, where the driving and driven streams are in contact. Nonuniformity in direction of flow and nonuniformity in pressure are of course ignored in our one-dimensional treatment. If these nonuniformities do not seriously modify the over-all picture, our model may still give useful results. At present, there is not much information establishing the limits of usefulness of the one-dimensional approach when applied to two interacting streams. However, the one-dimensional method has proved very useful for single streams, and we are more optimistic than Robison and Nelson about its usefulness for two or more streams.

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Comment on "Computer Analysis of Asymmetric Free Vibrations of Ring-Stiffened Orthotropic Shells of Revolution"

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In a recent article, Cohen[1] described a computer analysis of free vibrations of thin shells. Among the results he obtained were natural frequencies and modes for axisymmetric vibration of a 60° spherical dome, a case which had been studied previously by Kalnins.[2] Cohen found an extra frequency between the second and third frequencies for a fixed-hinged edge given in Table 1 of Kalnins[2] paper. The purpose of this Note is to make some remarks about this frequency and mode and to show how it can be found with very little computation by asymptotic theory.

The basis for these remarks is a paper by the writer,[3] in which approximate frequency conditions and modes were derived for spherical domes under a variety of edge conditions. For a fixed-hinged edge, the approximate frequency condition† is

$$[dP_{n_3}(\cos\phi)/d\phi]_{\phi=\psi}(2S(\psi) + \Lambda^{-1/2}\cot\psi\{(\tfrac{1}{8}) \times [S(\psi) - C(\psi)] + [(\tfrac{7}{8}) - \nu][S(\psi) + C(\psi)]\}) = \Lambda^{-1/2}P_{n_3}(\cos\psi)\,[\lambda_3 - (1-\nu) - (1-\nu^2)\Omega^2][C(\psi) - S(\psi)] \quad (1)$$

and the modes are for $\phi \neq 0$

$$w = P_{n_3}(\cos\phi) - (\tfrac{1}{2})P_{n_3}(\cos\psi)(\sin\psi/\sin\phi)^{1/2} \times \{[S(\phi)/S(\psi)] + e^{b(\phi-\psi)}\} \quad (2)$$

$$u/(1+\nu) = [\lambda_3 - (1-\nu) - (1-\nu^2)\Omega^2]^{-1}\, dP_{n_3}(\cos\phi)/d\phi + (\Lambda^{-1/2}/2) \times P_{n_3}(\cos\psi)(\sin\psi/\sin\phi)^{1/2}\{-[C(\phi)/S(\psi)] + e^{b(\phi-\psi)}\} \quad (3)$$

and for $\phi \approx 0$

$$w = P_{n_3}(\cos\phi) - (\Lambda^{1/4}/2)(\tfrac{1}{2}\pi\sin\psi)^{1/4}P_{n_3}(\cos\psi) \times J_0(\Lambda^{1/2}\phi)/S(\psi) \quad (4)$$

$$u/(1+\nu) = [\lambda_3 - 1 + \nu - (1-\nu^2)\Omega^2]^{-1}dP_{n_3}(\cos\phi)/d\phi + (\Lambda^{-1/4}/2)(\tfrac{1}{2}\pi\sin\psi)^{1/2}P_{n_3}(\cos\psi)J_1(\Lambda^{1/2}\phi)/S(\psi) \quad (5)$$

Here, u and w are the meridional and normal (outward) displacements; ϕ is the latitude coordinate ($\phi = 0$ is the pole, and $\phi = \psi$ the edge); and ν is Poisson's ratio. Also, if ω is the frequency, E Young's modulus, h, ρ, and R the thickness, density, and radius of the sphere, then

$$\Omega^2 = \omega^2R^2\rho/E$$

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† This is not quite the same as the frequency condition stated in Eq. (54) of the paper.[3] In obtaining the latter form, an additional approximation $S(\psi) = O(\Lambda^{-1/2})$ was introduced into the previous condition. That approximation is accurate only for the bending frequencies. Here we deal with membrane frequencies and thus must use the frequency condition in its pristine form (1).